Technical Paper:

Valuation of Credit Default Swaps

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1 Introduction

A credit default swap is a financial instrument which is used to protect a company from the default of a counterparty. It has the following property: the counter party of the credit default swap pays (1-R) at time of default - R is the recovery rate - for a fee s (the credit spread) - which is usually paid on a quarterly basis.
Credit default swap curves are curves, which are defined by maturities of credit default swaps and corresponding "fair" credit spreads - i.e. if a credit default swap with maturity T is entered today then the following equation has to be fulfilled by the corresponding credit spread (the value of the credit default swap is 0):

$$S\sum_{j=1}^{n} a(T_{j-1},T_j) * [D(T_j) * S(T_j) + \frac{1}{T_j - T_{j-1}} \int_{T_{j-1}}^{T_j} (t - T_{j-1}) D(t) f(t) dt]$$
$$= (1 - R) \int_{T_0}^{T} D(t) f(t) dt$$

with

- $h(t) \ldots$ Hazard Rate
- $S(t) = e^{\int_0^t h(s)ds} \ldots$Survival Probability
- $f(t) = h(t) * S(t) \ldots$density of default
- $a(T_{j-1}, T_j) \ldots$Daycount fraction from T_{j-1} to T_j
- $D(t) \ldots$ Risk free discount factor

Within the UnRisk PRICING Engine we assume the hazard rate to be constant between the given maturity dates of the credit default swap curve.

2 Dirty and clean value

The dirty value of a credit default swap is calculated by the use of the following algorithm:

1. Perform credit bootstrapping to generate the hazard rates from the given credit default swap curve.
2. Calculate the credit spread which is implied by the given maturity date of the credit default swap, the given interest rate curve and the given credit default swap curve. We call this implied credit spread sI.
3. The dirty value of the credit default swap is then given by the following formula (see the introduction)

$$\begin{aligned} \text{value} &= \sum_{j=1}^{n} (sI * a_1(T_{j-1}, T_j) - sX * a_2(T_{j-1}, T_j)) \\ &\quad * [D(T_j) * S(T_j) + \frac{1}{T_j - T_{j-1}} \int_{T_{j-1}}^{T_j} (t - T_{j-1}) D(t) f(t) dt] \end{aligned}$$

with

- $sX \ldots$ Credit Spread of the credit default swap
- $sI \ldots$ Implied Credit Spread of the credit default swap curve
- $h(t) \ldots$ Hazard Rate
- $S(t) = e^{\int_0^t h(s)ds} \ldots$Survival Probability
- $f(t) = h(t) * S(t) \ldots$density of default
- $a(T_{j-1}, T_j) \ldots$Daycount fraction from T_{j-1} to T_j
- $D(t) \ldots$ Risk free discount factor

a_1 and a_2 are the same if the settlement date does not lie between T_{j-1} and T_j. If the settlement date lies between T_{j-1} and T_j then a_1 is given as the daycount fraction accruing from settlement date to T_j and a_2 is given as the daycount fraction accruing from the previous payment date (or the start date) to T_j. The clean value is calculated by subtracting the daycount fraction accruing from the previous payment date (or the start date) to the settlement date from the $sX * a_2(T_{j-1}, T_j)$ term from above - from this it follows that the dirty value is smaller or equal than the clean value.